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Role of Eigenvectors in Aircraft Dynamics Optimization

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A multiobjective strategy to optimize aircraft flight dynamics is presented. Instead of using a classical approach where systematic methodologies are scarce to define a suitable configuration, a two-step procedure is proposed to guide the designer through an eigenstructure-assignment-based control law. First, an ideal model is found by means of a multiobjective optimization strategy that includes constraints both on eigenvalues and eigenvectors. The problem is solved through a hybrid procedure that combines genetic algorithms with deterministic local search methods. Second, the ideal model is approximated with a variable structure control system to improve the control system robustness. A comparison is shown with examples taken from the literature, where the effectiveness of the proposed methodology is demonstrated.

Introduction

BECAUSE of physical convenience and mathematical tractability, the definition of a suitable aircraft eigenstructure has often attracted researchers as a natural means to obtain desired flight dynamics behaviors. Eigenstructure-based control laws also have been successfully employed in real flight control systems, for instance, in the lateral stability augmentation system of Airbus A320 (Ref. 1).

Eigenstructure assignment by means of state or output feedback control has greatly evolved, especially over the last two decades, after Moore[2] first pointed out the full-state feedback power in completely specifying the closed loop eigenvalues and partially assigning the corresponding eigenvectors. Since then, numerous methods and algorithms have been developed to exploit those degrees of freedom fully and to guarantee good system performance. In the aerospace field, a considerable number of papers have focused on different aspects of the problem, either trying to improve the performance of the closed-loop dynamics or to minimize the required control effort.[3–8] Both of these conflicting design objectives play a fundamental role in aircraft dynamics optimization. Therefore, a tradeoff strategy must be suitably selected, and a multiobjective eigenstructure assignment problem arises naturally.

The concept of a generic multiple objective control problem that involves different types of performance indices is an old and very interesting idea. Most of the available literature concerning multiobjective eigenstructure assignment is dedicated to the problem of combining performance selection with system robustness. This problem has been studied from several different viewpoints: by orthogonalizing the eigenvectors[9] and by using the H_∞ norm,[10,11] the structured singular value,[12] and/or eigenvalue sensitivity.[13,14] All of these different methodologies have a common denominator, in that they are faced with the problem of how to change the design specifications to provide adequate stability robustness to the resulting feedback system. This is possible because, in most cases, the designer has a certain degree of freedom in making choices. For instance, according to flying qualities criteria, requirements are often given in terms of desired regions of the closed-loop eigenvalues and acceptable sets of eigenvectors shapes. However, while robustness is increased, the designer's degrees of freedom are reduced with serious consequences on the system performance. Following this line of reasoning, in a previous paper[15] we proposed a methodology that allows optimal selection of the eigensystem and, at the same time, to limit the maximum input demand on the control surfaces. Robustness considerations have been implicitly taken into account by formulating the problem in a linear quadratic framework. In that paper, the optimal solution is found by minimizing a quadratic performance index through gradient information on the objective function and constraints. Although excellent results have been obtained, in some cases it may be interesting to expand the designer's flexibility by choosing a more general performance index and including the eigenvectors in the optimization process. It turns out that performance improvements may be obtained by allowing eigenvectors to "vary" within specified constraints, instead of forcing them to assume prescribed values. This is an important point that has been overlooked in the available literature and is one of the contributions of the present paper. It should be clear that in this new context, a suitable optimization strategy must be used and the system robustness must be guaranteed. The first point is addressed with a hybrid procedure that combines genetic algorithms with deterministic local search methods. The second point is approached by designing a variable structure control (VSC) system that approximates the results of the optimization process and "recovers" the system robustness.

In recent years VSC systems with sliding modes have been proposed for many different aircraft and helicopter configurations.[16–19] VSC methodology is particularly interesting because the motion on the sliding surface is insensitive to plant parameter variations and external disturbances and is formally equivalent to an unforced system of lower order. In other words, VSC may be represented as a discontinuous controller that switches across a sliding surface in the state space. This surface attracts the system trajectory and the subsequent motion is constrained to remain in the sliding subspace.

In most cases the system under control is linear (at least in a neighborhood of the equilibrium condition), and the sliding surface is chosen to be an hyperplane. Under these assumptions the switching surface design problem can be translated into a design procedure that prescribes a linear full state feedback controller for a linear dynamic system. For these reasons, there are close connections between VSC systems and eigenstructure assignment procedures. However, a profound gap still exists between these two design techniques because, to date, no systematic approach has been proposed to combine VSC and eigenstructure design effectively. Indeed, in the available VSC literature, the sliding surface is typically designed regardless of how an eigensystem has been selected, and in particular, the eigensystem is usually chosen as one of the infinite acceptable configurations. Actually, important system improvements may be obtained by carefully combining VSC design and eigenstructure assignment methodologies. This claim will be apparent in the next sections and represents another important contribution of the present paper.

System Model and Eigenstructure Assignment

The aircraft dynamics are described by the following linear, time-invariant system:

$$\dot{x} = Ax + Bu \tag{1}$$

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$$y = Cx \quad (2)$$

where $x \in \mathbb{R}^n$ is the vector containing the states of the system, $u \in \mathbb{R}^m$ is the control input vector, $y \in \mathbb{R}^p$ is the vector of output signals, $A \in \mathbb{R}^{n\times n}$, $B \in \mathbb{R}^{n\times m}$, and $C \in \mathbb{R}^{p\times n}$. In the following we assume that 1) $p > m$, 2) the system is controllable and observable, and 3) the matrices B and C are of full rank.

Our aim is to find a suitable control law, in the form

$$u = Ky \quad (3)$$

where K is an $m \times p$ matrix, in such a way that the closed-loop system has a prescribed set of eigenvalues $\{\lambda_i^d\}$ and corresponding eigenvectors $\{v_i^d\}$. Eigenvalue/eigenvector pairs are solutions of the following equation:

$$(A + BKC)v_i = \lambda_i v_i \quad (4)$$

Actually, the closed-loop eigensystem cannot be completely specified. Instead, it is well known[20] that p closed loop eigenvalues can be arbitrarily assigned with m components of the corresponding eigenvectors. Accordingly, we assume that

$$\lambda_i^d = \lambda_i, \qquad i = 1, \ldots, p \quad (5)$$

Because not all of the components of a desired eigenvector v_i^d may be assigned, it is useful to define a reordering operator $\{\cdot\}^{R_i}$ as follows[21]:

$$\{v_i^d\}^{R_i} = \begin{bmatrix} l_i^d \\ d_i \end{bmatrix} \quad (6)$$

where l_i^d and d_i are vectors of specified and unspecified components of v_i^d. When

$$L_i \triangleq (\lambda_i I - A)^{-1} B \quad (7)$$

it has been shown by Andry et al.[21] that an achievable eigenvectors v_i^a is given by

$$v_i^a = L_i \tilde{L}_i^\dagger l_i^d \quad (8)$$

where the superscript dagger denotes the pseudoinverse and $\tilde{L}_i$ is obtained by applying the reordering operator to L_i, that is,

$$\{L_i\}^{R_i} = \begin{bmatrix} \tilde{L}_i \\ D_i \end{bmatrix} \quad (9)$$

When

$$V \triangleq \left[v_1^a, \ldots, v_p^a\right] \quad (10)$$

$$\Lambda \triangleq \mathrm{diag}(\lambda_1, \ldots, \lambda_p) \quad (11)$$

the control matrix K is obtained as

$$K = B^\dagger (V\Lambda - AV)(CV)^{-1} \quad (12)$$

Note that the algorithm could be slightly modified to deal with noninvertible matrices.[7,21]

Problem Statement and Solution

From an engineering standpoint, a complete closed-loop eigenstructure assignment is neither possible nor practical. Accordingly, we consider the problem of assigning r entries of the eigenvector matrix V and p eigenvalues. To this end, we first arrange those $p + r$ entries in a vector z and define

$$z \triangleq \begin{bmatrix} z_\lambda \\ z_v \end{bmatrix} \quad (13)$$

where

$$z_\lambda = \left[z_{\lambda_1}, \ldots, z_{\lambda_p}\right]^T, \qquad z_v = \left[z_{v_1}, \ldots, z_{v_r}\right]^T \quad (14)$$

In particular, as long as complex eigenvalues are concerned, we consider their natural frequency ω and damping ratio ζ. To improve the numerical accuracy of the optimization algorithm (to be described next), the components of vector z have been normalized with the corresponding maximum allowable value. For instance, when z_{λ_i} and z_{v_i} are the generic entries of z, complex eigenvalues are arranged as

$$z_{\lambda_i} = \omega_i \big/ \omega_i^{\max} \quad (15)$$

$$z_{\lambda_{i+1}} = \zeta_i \quad (16)$$

and real (negative) eigenvalues are treated as

$$z_{\lambda_i} = \lambda_i \big/ \lambda_i^{\min} \quad (17)$$

Also, it is assumed that eigenvectors have been normalized to 1 so that $|z_{v_i}| \le 1$. The optimal control law, whose structure is given by Eq. (3), is obtained by minimizing a nonnegative functional J in the form

$$J = \min_z \max_t f(z, t) \quad (18)$$

subject to

$$z_i^{\min} \le z_i \le z_i^{\max}, \qquad i = 1, \ldots, r + p \quad (19)$$

The choice of an adequate functional is obviously problem dependent and plays a crucial role in obtaining good results. In a previous paper,[15] we used a linear quadratic functional. In this case, we generalize the approach by introducing a more intuitive and flexible functional and a consequent new procedure for its minimization. We assume that

$$f(z, t) = \sum_i \frac{|y_i(t)|}{W_{y_i}} + \frac{\|K\|}{W_K} \quad (20)$$

where y_i is the ith component of the output vector defined by Eq. (2), $\|\cdot\|$ represents the Euclidean norm, and W_{y_i} and W_K are suitable weights. Note that y_i and K are related to z through Eqs. (1–4), (12), and (13).

A few remarks are in order. The summation term in Eq. (20) allows one to penalize the peak values of the output components. The last term, instead, is closely related to the required control effort. Also, note that the solution of the optimum J is not simple because the problem is not convex and is characterized by the presence of several local minima. In this context, an important drawback of classical gradient-based methods is their strong dependence on the initial configuration. Indeed, they generally stall in the local minimum closest to the starting point. To overcome this obstacle, stochastic global search techniques, not requiring the use of gradients, have been considered. In particular, genetic algorithms (GAs), which are based on an imitation of the elitist reproduction process of biological evolution (Goldberg[22]), appear particularly attractive in a complex context due to their robustness. Indeed, GAs can explore the entire allowable space, even if it is nonconvex or disjointed, and thus, they escape from local minima to search for a global solution. Generally speaking, evolutionary algorithms as GAs require a large number of objective function evaluations: Accordingly, the high time-consuming computational effort is an important limitation. Actually, the lack of supplementary information about the derivatives (which is characteristic of GAs) does not permit a fast convergence in the nearness of the optimum. To improve the computational efficiency, several routes can be followed, for instance, coupling GAs with neural networks[23] or using them in parallel computer architectures.[24] In our case, a natural choice is to consider a hybrid procedure that combines GAs with deterministic local search methods. In particular, in this paper we use a two-stage strategy that splits the optimization procedure in two steps: First, the genetic algorithms explore a large search space and localize the global optimum region. Then a deterministic gradient-based method is employed to accurately reach the optimum. More complex hybridization have also been developed (for instance, Dulikravich et al.,[25] where four different optimizers are used), but the described choice seems to offer a good tradeoff between reliability and complexity. Its effectiveness is shown in the example discussed in the next section.

Case Study: Pitch Pointing Control System Design

To better explain the methodology, we consider the design of a pitch pointing flight control system. For comparative purposes in what follows we use an example originally proposed by Sobel and Shapiro.[26] The plant under consideration is the short-period approximation of an AFTI/F-16 model. The flight condition corresponds to an altitude $h = 3000$ ft and a Mach number $M = 0.6$. The equations of motion are in the form of Eqs. (1) and (2) with

$$\boldsymbol{x} = [\alpha \quad q \quad \gamma \quad \delta_e \quad \delta_f]^T \tag{21}$$

$$\boldsymbol{u} = \left[\delta_{e_c} \quad \delta_{f_c}\right]^T \tag{22}$$

$$\boldsymbol{y} = [q \quad n_z \quad \gamma \quad \delta_e \quad \delta_f]^T \tag{23}$$

where α is the angle of attack, q is the pitch rate, γ is the flight-path angle, δ_e and δ_f are elevator and flaperon deflections, δ_{e_c} and δ_{f_c} are elevator and flaperon command deflections, and n_z is the normal acceleration at the pilot's station. Matrices A, B, and C are reported in Appendix along with open-loop eigenvalues. Note that the system is unstable due to the presence of an unstable short-period mode.

In pitch pointing control design, the objective is to allow pitch attitude control while maintaining constant flight-path angle. This can be accomplished by a suitable choice of eigenvectors: In particular it is important to decouple the short period and flight-path mode. This, in turn, implies that pitch rate and flight-path angle eigenvectors should be near orthogonal.

We first note that the system at hand is both controllable and observable, and because there are five outputs and two control inputs, it is possible to assign all of the closed-loop poles (indeed $p = n$) and two entries of each corresponding eigenvector.[20]

The approach by Sobel and Shapiro[26] consists of assigning the exact position of the closed-loop poles and the desired "shape" of closed-loop eigenvectors. With the flying the qualities requirements in mind, they choose the following vector $\boldsymbol{\lambda}_c$ of closed-loop pole locations:

$$\boldsymbol{\lambda}_c = [-5.6 \pm j\,4.2 \quad -1.0 \quad -19.0 \quad -19.5]^T \tag{24}$$

Note that the complex poles correspond to a short-period mode with natural frequency and damping ratio equal to 7 rad/s and 0.8, respectively. The third pole represents the pitch-attitude mode, and the last two eigenvalues are elevator and flaperon modes. Some entries of the desired eigenvectors are also exactly specified, and the corresponding matrix gain K of closed-loop control law is evaluated according to the already summarized methodology [see Eqs. (3–12)]. In their example, an exact decoupling between short-period and flight-path mode is obtained with the following desired eigenvector matrix:

$$V_d = \begin{bmatrix} 1 & * & 0 & * & * \\ * & 1 & 0 & * & * \\ 0 & 0 & 1 & * & * \\ * & * & * & 1 & * \\ * & * & * & * & 1 \end{bmatrix} \tag{25}$$

where the asterisk refers to an unspecified component. Note that the first two columns of V_d represent real and imaginary parts of the short-period (complex) mode.

The major problem with this procedure is that the designer has no guidelines when a modification of choices is necessary, for instance, if an unacceptable input demand is required. Also, information is not easily available to quantify how good the design is or if a better solution exists.

For these reasons, in the present paper we propose a different approach. First, recall that flying quality requirements do not force an exact pole location, but, rather, they allow one to select the closed-loop eigenvalues by means of constraints in the form of Eq. (19). More precisely, z_i is in the form of Eq. (15) and (16) or (17) according to whether the eigenvalue is complex or real. Also, the assignable eigenvector entries do not need to be precisely defined: It is possible, for instance, define "to what extent" pitch rate and flight-path angle eigenvectors are orthogonal by allowing their values to vary within prescribed limits [once again in the form of Eq. (19)]. This suggests substituting the desired V_d matrix of Eq. (25) with the following one:

$$\tilde{V}_d = \begin{bmatrix} z_{v_1} & z_{v_3} & z_{v_5} & * & * \\ * & * & z_{v_6} & * & * \\ z_{v_2} & z_{v_4} & * & * & * \\ * & * & * & 1 & * \\ * & * & * & * & 1 \end{bmatrix} \tag{26}$$

where $|z_{v_1}|$ and $|z_{v_3}|$ should be "big" (near unity) and the others ($|z_{v_2}|$, $|z_{v_4}|$, $|z_{v_5}|$, and $|z_{v_6}|$) should be "small" (near zero). Note that no more than two entries of each eigenvector have been specified because there are only two control inputs.

With these preliminaries, the control law is obtained by solving Eqs. (18) and (19). In this case the functional to minimize is assumed to be

$$f(z, t) = \frac{|\gamma(t)|}{W_\gamma} + \frac{|\delta_e(t)|}{W_{\delta_e}} + \frac{|\delta_f(t)|}{W_{\delta_f}} + \frac{\|K\|}{W_K} + \frac{|\delta_e(t) - \delta_f(t)|}{W_d} \tag{27}$$

Note that weighting δ_e and δ_f peaks (by means of W_{δ_e} and W_{δ_f}) and suitably choosing W_K allows one to take the required command energy under control. Also, the last term in Eq. (27) has been introduced to "equalize" the control effort of elevator and flaperon. Indeed, it is physically clear that the two control inputs must rotate in opposite directions to achieve good pitch pointing behavior of aircraft.

Although pitch pointing design is our primary objective, different strategies may be used to guide the designer in the selection of the weights in Eq. (27). Here we consider the following two cases:

1) The solution should favor the minimization of control effort.

2) The solution should favor the minimization of flight-path angle while allowing pitch attitude control.

For both cases, the same constraints on eigenvectors are $0.1 \le z_1 \le 1$, $-0.01 \le z_2 \le 0.01$, $-1 \le z_3 \le -0.1$, $-0.01 \le z_4 \le 0.01$, $-0.1 \le z_5 \le 0.1$, and $-0.1 \le z_6 \le 0.1$. The constraints on eigenvalues for both cases are shown in Table 1. The weights used in Eq. (27) are shown in Table 2. Note that cases 1 and 2 simply differ in the value of W_γ. The results obtained from Eq. (27) are summarized in Tables 3 and 4. Figures 1 and 2 show simulation results.

The tradeoff between pitch pointing effectiveness and input control effort is apparent.

In case 1 a lesser pitch pointing effectiveness (with respect to Sobel and Shapiro[26]) is compensated by a significant saving of input control demand. In particular, the flaperon peak is reduced by nearly 70% with a concurrent substantial decrease in the norm of the matrix gain. Note that the the maximum absolute value of elevator and flaperon peak values are near identical.

In case 2, on the other hand, the maximum peak of flight-path angle is reduced at the cost of an increase of elevator and flaperon deflection.

The two cases are representative of different design strategies and clearly demonstrate the flexibility of the proposed methodology. Note that in a standard eigenstructure assignment problem the

Table 1 Eigenvalue constraints

Mode	Constraint	
Short period	$3 \le \omega \le 10$	$0.35 \le \zeta \le 1$
Pitch attitude	$-5 \le \lambda \le -1$	
Elevator	$-20 \le \lambda \le -18$	
Flaperon	$-20 \le \lambda \le -18$	

Table 2 Weights used in the optimization process

Case	W_γ	W_{δ_e}	W_{δ_f}	W_K	W_d
1	0.01	0.5	0.5	10	0.1
2	0.01/30	0.5	0.5	10	0.1

Table 3 Closed loop eigenvalues and corresponding gain matrices compared to Sobel and Shapiro[26]

Case	Eigenvalues	Gain matrix K
Sobel and Shapiro[26]	$[-5.6 \pm j\,4.2 \quad -1.0 \quad -19.0 \quad -19.5]$	$\begin{bmatrix} -0.931 & -0.149 & -3.25 & -0.153 & 0.747 \\ 0.954 & 0.210 & 6.10 & 0.537 & -1.04 \end{bmatrix}$
1	$[-9.04 \pm j\,0.33 \quad -1.0 \quad -20.0 \quad -18.0]$	$\begin{bmatrix} 1.456 & 0.194 & 2.786 & -0.093 & -1.018 \\ -1.588 & -0.196 & -4.521 & 0.201 & 1.111 \end{bmatrix}$
2	$[-9.99 \pm j\,0.12 \quad -1.69 \quad -18.0 \quad -20.0]$	$\begin{bmatrix} 1.778 & 0.220 & -2.931 & -0.282 & -1.228 \\ -3.603 & -0.468 & -1.929 & 0.559 & 2.495 \end{bmatrix}$

Table 4 Comparison between norms of K matrix and peak values (in degrees) of elevator, flaperon, and flight-path angle, relative to an initial angle of attack of 1 deg

Case	$\|K\|$	δ_e	δ_f	γ
Sobel and Shapiro[26]	7.17	4.62	−7.67	0.06
1	5.92	4.45	−4.49	0.24
2	4.99	5.24	−10.58	0.036

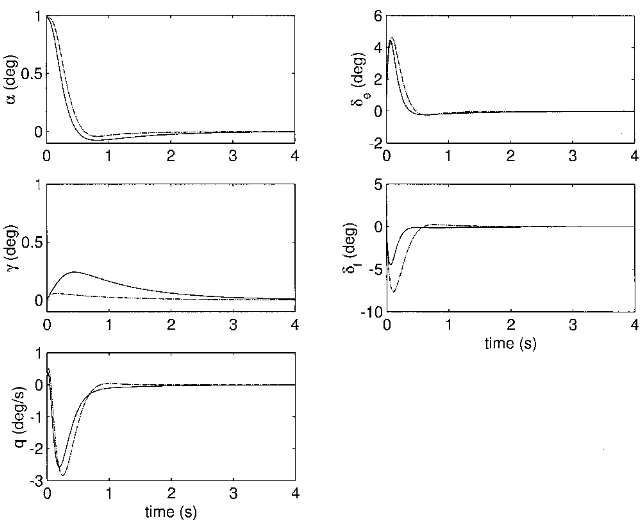

Fig. 1 Comparison between time histories of closed-loop systems relative to an initial angle of attack of 1 deg: ——, case 1 and –·–·, Sobel and Shapiro.[26]

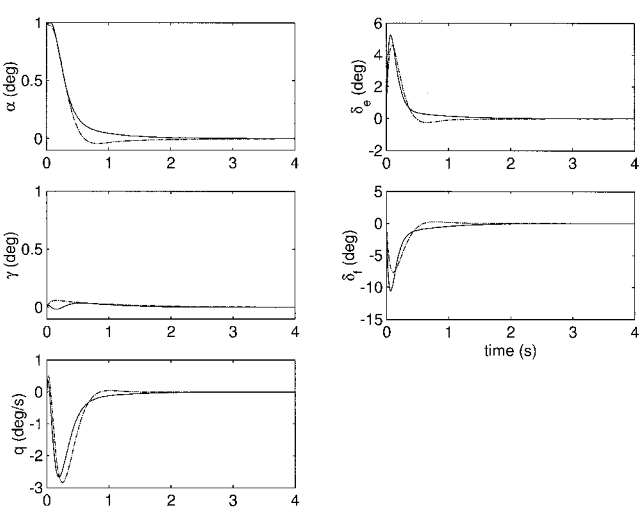

Fig. 2 Comparison between time histories of closed-loop systems relative to an initial angle of attack of 1 deg: ——, case 2 and –·–·, Sobel and Shapiro.[26]

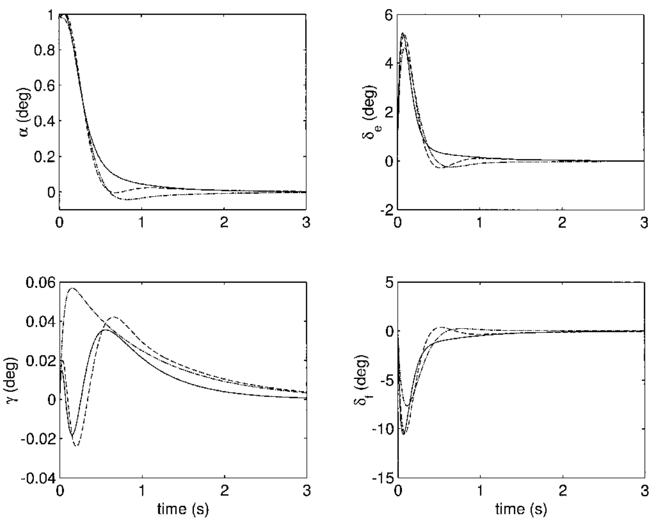

Fig. 3 Comparison between time histories of closed loop systems relative to an initial angle of attack of 1 deg: ——, case 2; –·–·, Sobel and Shapiro[26]; and – – –, case 3.

designer is "blind" with respect to the results. How a different choice of the desired eigenvalues and eigenvectors will affect the final design cannot be predicted. The two case studies indicate that the main differences and advantages of the proposed methodology may be summarized in three points. First, the conflicting requirements between pitch pointing effectiveness and input demand effort are met by a suitable choice of the weights in the functional. Second, the designer has a quantitative information about the limits of performance of the controller and can establish if and how to modify the design. Third, the role of the eigenvectors is fully exploited. These three points are of a fundamental nature and are valid in a generic eigenstructure assignment problem. From this viewpoint, the two case studies give important information about the potential of the approach when compared to a classical design.

At this point one may wonder about the relative importance of an optimal choice of eigenvectors. To obtain meaningful results, it is useful to compare the reference case (by Sobel and Shapiro[26]) with case 2 and with an intermediate design (referred to as case 3 in the sequel). The latter differs from the reference case only for the choice of eigenvectors. More precisely, in case 3 the optimization process is performed assuming the same eigenvalues of the reference case and only optimizing the eigenvectors. (Note that the weights used in Eq. (27) are those of case 2). The simulation results are shown in Fig. 3. Note that eigenvector optimization may produce significant improvements over the reference design. (Indeed, the peak value of flight-path angle is reduced by nearly 40% in case 3 as compared to the design of Sobel and Shapiro.[26]

Sliding Mode Control

In the preceding sections a methodology has been described to obtain easily an optimal control law from the viewpoint of aircraft dynamic response. An important question is the robustness of the resulting design against parameter variations, model uncertainties, etc. Instead of sacrificing some degree of freedom to include the robustness issue into the design process, we propose using a VSC system that approximates the result of the optimization process. In this way, the optimized model may be considered as an ideal reference model, and the VSC system is used to recover its robustness. To this end, referring to the aircraft dynamics described in Eq. (1), consider the switching function

$$s(t) = Sx(t) \tag{28}$$

where $S \in \mathbb{R}^{m \times n}$ is a full rank matrix, and define the switching hyperplane $Sx = 0$. A sliding motion takes place on the hyperplane provided there exists a time t_s, for all $t \geq t_s$ such that

$$s(t) = Sx(t) = 0 \tag{29}$$

Under the assumption that SB is nonsingular, it can be shown[27] that the sliding motion results in a control independent free motion described by

$$\dot{x} = [I - B(SB)^{-1}S]Ax = \hat{A}x \tag{30}$$

The sliding motion is of reduced order and, indeed, matrix $\hat{A}$ has at most $n - m$ nonzero eigenvalues. Accordingly, an $(n - m)$th-order set of equations describing the sliding motion is obtained by eliminating m state variables through Eq. (29). Following Zinober,[27] this is accomplished through the transformation

$$w = \begin{bmatrix} w_1 \\ w_2 \end{bmatrix} = Tx \tag{31}$$

where $w_1 \in \mathbb{R}^{n-m}$, $w_2 \in \mathbb{R}^{m}$, and $T \in \mathbb{R}^{n \times n}$ is an orthogonal matrix such that

$$TB = \begin{bmatrix} 0 \\ B_2 \end{bmatrix} \tag{32}$$

$$TAT^T = \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix} \tag{33}$$

$$ST^T = [S_1 \quad S_2] \tag{34}$$

with $B_2 \in \mathbb{R}^{m \times m}$, $S_1 \in \mathbb{R}^{m \times (n-m)}$, and $S_2 \in \mathbb{R}^{m \times m}$.

The nominal system (1) is equivalent to

$$\dot{w}_1 = A_{11}w_1 + A_{12}w_2 \tag{35}$$

$$\dot{w}_2 = A_{21}w_1 + A_{22}w_2 + B_2u \tag{36}$$

During a sliding motion, for all $t \geq t_s$, Eq. (29) gives

$$S_1w_1(t) + S_2w_2(t) = 0 \tag{37}$$

from which, assuming that S_2 is nonsingular, one has

$$w_2 = -S_2^{-1}S_1w_1 \tag{38}$$

Finally, when

$$K_S \triangleq S_2^{-1}S_1 \tag{39}$$

the motion of the $(n - m)$th-order system is

$$\dot{w}_1 = (A_{11} - A_{12}K_S)w_1 \tag{40}$$

This result implies that the hyperplane design is equivalent to a state feedback control problem for the system (35) where $w_2(t)$ plays the role of the control vector.

Note that the switching function is known as soon as K_S is found. Several techniques have been proposed for the design of the feedback matrix. A particularly useful approach involves the minimization of a quadratic performance index, in the form[28]

$$J_S = \frac{1}{2}\int_{t_s}^{\infty} x^T(t)Qx(t)\,\mathrm{d}t \tag{41}$$

where Q is a symmetric and positive definite matrix. Applying the transformation (31), one has

$$TQT^T \triangleq \begin{bmatrix} Q_{11} & Q_{12} \\ Q_{12}^T & Q_{22} \end{bmatrix} \tag{42}$$

and Eq. (41) is equivalent to

$$J_S = \frac{1}{2}\int_{t_s}^{\infty} \left(w_1^TQ_{11}w_1 + 2w_1^TQ_{12}w_2 + w_2^TQ_{22}w_2\right)\mathrm{d}t \tag{43}$$

Standard software is available to minimize J_S (e.g., the MATLAB® function lqr). In particular, the optimal gain K_S, where the feedback control law

$$w_2 = -K_Sw_1 \tag{44}$$

minimizes Eq. (43) subject to system (35), is found to be[29]

$$K_S = Q_{22}^{-1}\left(A_{12}^TP + Q_{12}^T\right) \tag{45}$$

where P is the solution of the algebraic Riccati equation

$$A_{11}^TP + PA_{11} - (PA_{12} + Q_{12})Q_{22}^{-1}\left(A_{12}^TP + Q_{12}^T\right) + Q_{11} = 0 \tag{46}$$

To summarize, the choice of a matrix Q implies a corresponding definition of the switching hyperplane. Many different VSC structures exist in the literature, but in the following we consider the so called "unit vector" approach. Details are omitted here, and the interested reader is referred to Edwards and Spurgeon.[30] Essentially, the control law triggering the sliding motion comprises a linear component $u_l(t)$ and a nonlinear component $u_n(t)$, that is,

$$u(t) = u_l(t) + u_n(t) \tag{47}$$

In particular, the linear component is the solution of

$$u_l(t) = -(SB)^{-1}(SA - \Phi S)x(t) \tag{48}$$

where $\Phi \in \mathbb{R}^{m \times m}$ is any stable design matrix. Also, for $s(t) \neq 0$,

$$u_n(t) = -\rho(SB)^{-1}[Rs(t)/\|Rs(t)\|] \tag{49}$$

where $R \in \mathbb{R}^{m \times m}$ is a symmetric positive definite matrix satisfying the Lyapunov equation,

$$R\Phi + \Phi^TR = -I \tag{50}$$

and the positive scalar ρ depends only on the magnitude of the model uncertainty. This control law guarantees that the switching surface is reached in a finite time, and once the sliding motion is attained, it is independent of the uncertainty. A detailed description of the design methodology is given in the next section.

Case Study: Pitch Pointing Control Revisited

To apply the VSC design methodology we need to choose a reference model. Suppose an optimal aircraft dynamics model has been found according to the guidelines of the pitch pointing control example. It is natural to consider the optimized model as the reference one. In particular, in the following we adopt the earlier defined case 1 as the ideal reference model. From the preceding discussion, it is clear that the reference model is the output of the optimization process and is the result of the designer's choices in terms of conflicting requirements. In this context, case 1 is used as a possible reference design to validate the proposed approach.

According to the VSC methodology, we essentially need to choose the Q matrix only. In other words, the matrix Φ [see Eq. (48)] and the scalar ρ are taken fixed, and to simplify the problem, Q is taken diagonal. Because Φ is strictly connected to the actuator bandwidth we assume that

$$\Phi = -\begin{bmatrix} 20 & 0 \\ 0 & 20 \end{bmatrix}$$

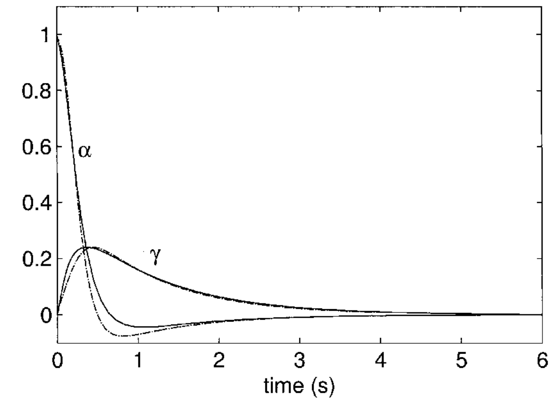

Fig. 4 Comparison between time histories of closed-loop systems relative to an initial angle of attack of 1 deg: ——, sliding mode control and –·–·, case 1.

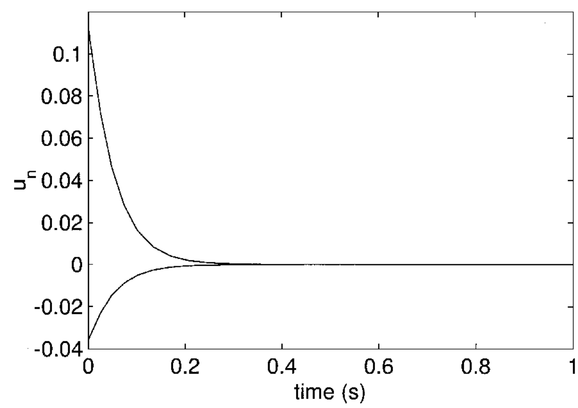

Fig. 5 Nonlinear part of the control law $u_n(t)$, showing the sliding mode engagement.

and that $\rho = 1$. The choice of matrix Q is easily accomplished by "forcing" the aircraft dynamics, controlled through Eq. (47), to follow the ideal model. This problem, in turn, is solved by minimizing the functional

$$J_Q = \int_0^T (\gamma - \gamma_m)^2 \, dt \tag{51}$$

where γ and γ_m represent the actual and ideal (model-based) flight-path angle and T is a simulation time on the order of the closed-loop dominant pole of the system. A simple GA is sufficient to solve the preceding problem with reasonable approximation. The result is

$$Q = \text{diag}(40 \quad 1 \quad 450 \quad 50 \quad 16) \tag{52}$$

where diag (·) means a diagonal matrix. With this choice, the VSC law is completely defined, and a comparison with the ideal reference model may be established through simulation. This has been done, and the results are shown in Fig. 4. Note that the VSC-based simulation closely follows that of the ideal design, thus indicating a good approximation with the reference case. Other interesting information is given by the time that the VSC system needs before a sliding motion takes place. This may be obtained by considering when the nonlinear part $\boldsymbol{u}_n(t)$ of the control law is negligible. Figure 5 shows that the sliding motion takes place in near 0.35 s. This is a sufficiently fast dynamics when compared to that of the aircraft.

Conclusions

Eigenstructure assignment is a powerful and physically significant way to optimize easily aircraft flight dynamics. In this paper, a new and consistent methodology has been described to guide the designer toward the synthesis of the control law. Basically, the proposed approach is divided in two separate steps.

First an optimal design (from the viewpoint of control matrix gains) is obtained by solving a multiobjective minimax problem. Although the problem is not convex, good results are obtained by means of a hybrid procedure combining GAs with deterministic local search methods. In this first phase, the designer specifies goals and constraints as a set of well-defined computational criteria, taking into account the various conflicting design objectives. These include flight and/or ride qualities criteria, maximum deflection and rate of deflection of the input commands, bandwidth, etc.

Second, the design robustness is recovered by means of a VSC system. Note that the two steps are strictly connected because the optimized model is used as a reference during the design of the VSC system.

Various advantages of the proposed approach over existing methodologies are as follows:

1) The designer easily gets a reference model from which useful information is derived to quantify the quality of different control laws.

2) Physically intuitive rules guide the designer in choosing the weights that ultimately define the optimal configuration.

3) The maximum allowed degree of freedom is obtained, and the fundamental role of eigenvectors is fully exploited.

4) A systematic guide is established to the design of the VSC system.

The whole methodology is simple to handle and may be effectively used to provide the designer with quick and effective solutions. A detailed case study has been presented to validate the proposed approach.

Appendix: Aircraft Data

The open-loop aircraft dynamics are described by Eqs. (1) and (2), where matrices A, B and C are given by

$$A = \begin{bmatrix} -1.341 & 0.9933 & 0 & -0.1689 & -0.2518 \\ 43.223 & -0.8693 & 0 & -17.251 & -1.5766 \\ 1.341 & 0.0067 & 0 & 0.1689 & 0.2518 \\ 0 & 0 & 0 & -20 & 0 \\ 0 & 0 & 0 & 0 & -20 \end{bmatrix}$$

$$B = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 20 & 0 \\ 0 & 20 \end{bmatrix}$$

$$C = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 \\ 47.76 & -0.268 & 0 & -4.56 & 4.45 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

Open-loop poles (eigenvalues of matrix A) are as follows.

Unstable short-period mode:

$$\lambda_1 = -7.662, \qquad \lambda_2 = 5.452$$

Pitch-attitude mode:

$$\lambda_3 = 0$$

Elevator-actuator mode:

$$\lambda_4 = -20$$

Flaperon-actuator mode:

$$\lambda_5 = -20$$

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